Life of Fred®

Geometry

Expanded Edition

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Note to Students

One day in the life of Fred. A Thursday just after his sixth birthday. In the few hours of that one day, you will experience a full course in geometry. Everything's here: the formulas, the definitions, the theorems, the proofs, and the constructions.

WHAT YOU'LL NEED TO HAVE

You will need at least one year of high school algebra. (*Life of Fred: Beginning Algebra Expanded Edition*)

In this geometry book for example, on p. 26 we will go from

$$y + y = z \quad \text{to} \quad y = \tfrac{1}{2} z.$$

It is preferable, however, that you have completed the two years of high school algebra. (*Life of Fred: Beginning Algebra Expanded Edition* and *Life of Fred: Advanced Algebra Expanded Edition*)

Most schools stick geometry between the two years of algebra—beginning algebra, geometry, advanced algebra—but there are a couple of reasons why this is not the best approach.

First, when you stick geometry between the two algebra courses, you will have a whole year to forget beginning algebra. Taking advanced algebra right after beginning algebra keeps the algebra fresh.

Second, the heart of geometry is learning how to do proofs. This requires an "older mind" than the mechanical stuff in the algebra courses. A person's brain develops in stages. Most three-year-olds don't enjoy quiz shows on television. Most ten-year-olds are not "interested" in the opposite sex.

Here is the better sequence of high school math courses:

beginning algebra
advanced algebra
Geometry
trig.

Geometry is one course that is different from all the rest. In the other courses, the emphasis is on calculating, manipulating and computing answers.

In arithmetic, you find 6% of $1200.

In beginning algebra, you solve $\frac{x^2}{x+2} = \frac{8}{3}$

In advanced algebra, you use logs to find the answer to: *If my waistline grows by 2% each year, how long will it be before my waist is one-third larger than it is now?* (Taken from a Cities problem at the end of chapter three in *Life of Fred: Advanced Algebra Expanded Edition*.)

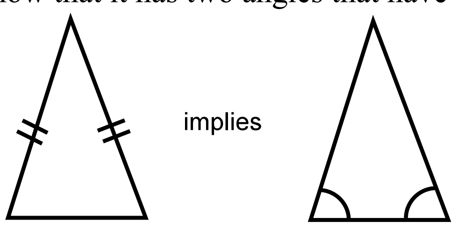
In trigonometry, you find the five different answers to $x^5 = 1$.

And then in calculus, you find the arc length of the curve $y^2 = 4x^3$ from $x = 0$ to $x = 2$. [From a Cities problem at the end of chapter 17 in *LOF: Calculus*.]

The answer, in case you're wondering, is $(2/27)(19^{3/2} - 1)$.

And then in linear algebra, which follows calculus, you will solve systems of many equations with many unknowns.

In contrast, in geometry there are proofs to be created. It is much more like solving puzzles than grinding out numerical answers. For example, if you start out with a triangle that has two sides of equal length, you are asked to show that it has two angles that have the same size.

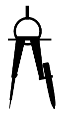
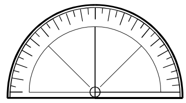


There are at least four different ways to prove that this is true. The proof that you create may be different than someone else's. Things in geometry are much more creative than in the other courses. It was because of the fun I experienced in geometry that I decided to become a mathematician.

The surprising (and delightful) truth is that geometry is much more representative of mathematics than are arithmetic, beginning algebra, advanced algebra, trig, or calculus. Once you get beyond all the "number stuff" of those courses, math becomes a playground like geometry. On page 216 I describe eight (of the many) math courses that you can take as a college student after calculus. All of them have the can-you-find-a-proof spirit that geometry has.

THE USUAL SUPPLIES FOR GEOMETRY

Besides a bit of algebra, you'll need the usual supplies for geometry:

✓ paper and a pencil
✓ a ruler
✓ a compass  (used in chapter 11 when we do constructions)
✓ maybe a protractor  (to measure angles)
✓ maybe your old hand-held calculator with +, –, ×, ÷, and $\sqrt{\ }$ keys on it.

The paper, pencil and ruler you probably already have. A compass may cost you a dollar or two.

If you're super rich and you want to blow another buck for a protractor, that's your choice. You won't really need one for this course.

WHAT YOU'LL NEED TO DO

Throughout this book are sections called *Your Turn to Play*, which are opportunities for you to interact with the geometry. Complete solutions are given for all the problems in the *Your Turn to Play* sections.

If you really want a solid grounding in geometry, just reading the problems and then just reading the solutions in the *Your Turn to Play* sections without working them out for yourself really won't work.

Just reading the questions and then just reading the answers in the *Your Turn to Play* sections is the easy way—but the material won't stick in your memory.

You have to haul out a sheet of paper and work out each problem on that sheet—and then look at the answer.

Some students have confessed that they just can't help "accidentally" looking at the answers to the *Your Turn to Play* questions that are given right below the questions. This is a question of dealing with temptation. The simple solution is to put a piece of paper over the answers so that you won't accidentally see them.

But what, you the reader ask, ***if the paper slips and I accidentally see the answer?***

Then hire some nasty 250-pound man who will make sure your paper doesn't slip.

At the end of each chapter are six sets of questions, each set named after a city in the United States. You may not have heard of some of the cities such as Elmira or Parkdale, but they all really exist. All of the answers are supplied right after the questions.

WHAT THIS BOOK CONTAINS

This book has *all* of high school geometry. Look over the Table of Contents (and the index) and you will find that it's all there.

In addition to the material for the average student there are six "Other Worlds" chapters (which are chapters 5½, 7½, 8½, 11½, 12½ and 13½) that offer you an ultra-complete honors course in geometry. For example, chapter 8½ is an introduction to symbolic logic. Chapter 11½ covers non-Euclidean geometry. Chapter 13½ deals with flawless, modern geometry. (Traditional Euclidean geometry has flaws in it.)

Near the back of the book, on p. 533, is the **A.R.T.** section (**A**ll **R**eorganized **T**ogether). The **A.R.T.** section lists all the theorems, definitions, and postulates in the order they are encountered in this book.

And *Life of Fred: Geometry Expanded Edition* tells the tale of one of the greatest love stories in world literature.

Contents

Chapter One

Points and Lines

Fred's two little eyes popped open. It was several minutes before dawn on an early spring morning. He awoke with a smile. There were so many things for which to be grateful. He ticked them off in his mind one by one.

✔ He was home, safe and sound.

✔ It was Thursday—one of his seven favorite days of the week.

✔ He had received a wonderful pet llama at his sixth birthday party last night.

✔ His math teaching position at KITTENS University.

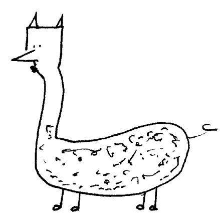
What a wonderful life! was his morning prayer. He stood up and stretched to his full height of thirty-six inches. Fred's home for the last five years or so was his office on the third floor of the math building at the university. By most standards he had been quite young when he first came to KITTENS.

Years ago, when he had arrived at the school, they had assigned him his office. He had never had a room of his own before. He was so tuckered out from all the newness in his life (a new job, a new state to live in, a new home) that he had just closed the door to his office and found a nice cozy spot (under his desk) and had taken a nap.

And every night since then, that is where he slept.

He had the world's shortest commute to work. And no expenses for an apartment or a car.

"Good morning Lambda!" he said to his llama. He had named her Lambda in honor of the Greek letter lambda (λ). He enjoyed the alliteration of "Lambda the llama."



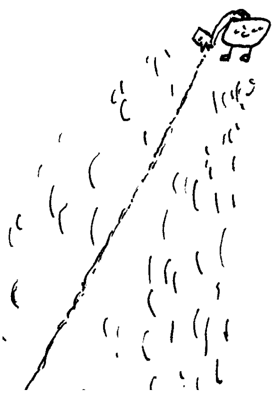
λ

She was busy chewing on the wooden fence that he had erected in his office.

"Would you like to get out and get some exercise with me?" he continued. Fred hopped into his jogging clothes. The pair headed down the two flights of stairs and out into an icy Kansas morning.

Fred was worried that his new pet wouldn't be able to keep up with him as he jogged. He had been jogging for years and running 15 or 20 miles was nothing for him. Fred's fears, however, were unfounded. His six-foot-tall llama had no trouble matching the pace of Fred's little legs.

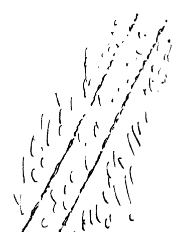
In fact, after a few minutes, Lambda spotted the new bocci ball lawn and raced ahead to enjoy some breakfast. By the time Fred caught up to her, she had mowed a straight line right across the lawn. Fred's little snacks that he had given her last night had left her hungry.



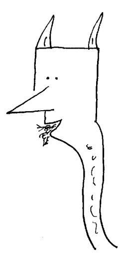
Oh no! Fred thought. Larry is gonna be mad. He put a lot of effort into that lawn. The international bocci ball tournament is scheduled to be here next week.

"Lambda, please come here. You're not supposed to be on Mr. Wistrom's lawn."

She, being a good llama, obeyed. On her way back to Fred she munched a second line in the grass. Fred loved his pet and didn't know that he was supposed to discipline her. Besides, he thought to himself those are such nice parallel lines.



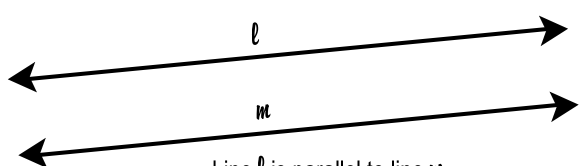
Parallel lines



Who could discipline such a lovely creature?

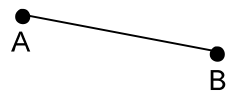
Actually, to be perfectly accurate, those are line segments he corrected himself. A line segment is just part of a line. It has two endpoints. A line is infinitely long in both directions. Anybody who's studied geometry knows that. Fred enjoyed the precision that mathematical language afforded him.

When Fred drew lines on the blackboard in his geometry class, he'd put arrows on ends to indicate that the lines went on forever. He labeled lines with lower case letters.



Line ℓ is parallel to line m

To draw a line segment was easy. You didn't need any arrows.

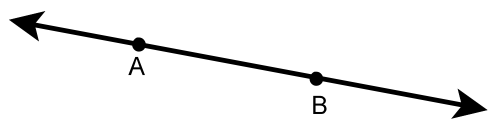


Line segment $\overline{AB}$ with endpoints A and B

Points like A and B are written with capital letters, and lines like ℓ and m are written with lower case letters.

$\overline{AB}$ is the notation for the line segment with endpoints A and B.

$\overleftrightarrow{AB}$ is the notation for the line which contains A and B.



Line $\overleftrightarrow{AB}$ which passes through A and B

And just to make things complete: AB is the *distance* between points A and B. That makes AB a number (like six feet) whereas $\overline{AB}$ and $\overleftrightarrow{AB}$ are geometrical objects (a segment and a line).

Your Turn to Play

1. Line segments (sometimes called segments for short) can come in lots of different lengths. Name a number that *couldn't* be a length of a line segment. (Please try to figure out the answer before you look at the solution furnished below.)

2. If AB = 4 and AC = 4, does that mean that A, B and C all lie on the same line?

3. If points D, E and F are collinear with E between D and F, it would *not* be right to say that $\overline{DE} + \overline{EF} = \overline{DF}$. Why not?

4. (A tougher question) If GH + HI = GI, must it be true that:

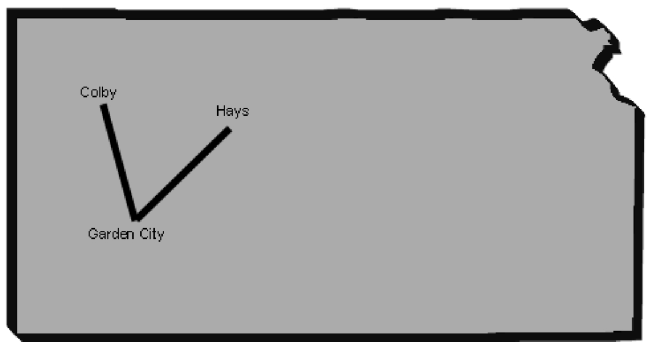
a) points G, H and I are collinear? b) H is between G and I?

The way to figure out the answer to this question is to get out a piece of paper and draw three points so that the distance from G to H plus the distance from H to I is equal to the distance from G to I. Be convinced in your own mind what the answers to questions a) and b) are before you look at the solutions below. You will learn very little by just reading the questions and then glancing at the answers.

. . . . COMPLETE SOLUTIONS . . . .

1. If you phone a travel agency and ask how far it is between San Francisco and Yosemite National Park, they might say something like 209 miles. If you phone them the next day and ask them how far it is from Yosemite National Park to San Francisco, they would again say 209 miles. The distance between two points is never negative. So the answer to question 1 might be something like –5 or –978267 or $-\pi$ or $-\sqrt{7}$. It could be zero since the distance from San Francisco to San Francisco is zero. In symbols, it's always true that AA = 0.

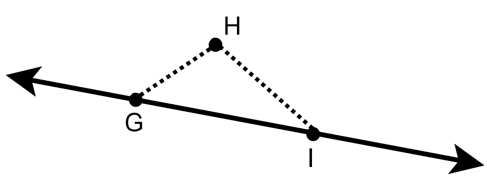
2. On this map of Kansas, the distance from Garden City to Colby is about the same as the distance from Garden City to Hays. Those three cities are not **collinear**. (Collinear = lie on the same line.)



3. It is true that DE + EF = DF (which states that the distance from D to E plus the distance from E to F is equal to the distance from D to F). We can add numbers together. But $\overline{DE}$ isn't a number. It's a line segment which is a geometrical object. The only thing we know how to add are numbers. Could you add Martin Luther and pizza? Could you divide the Red Cross by a stop sign?

Similarly, you can't draw AB since AB is a number (the distance between points A and B).

4. Suppose for a moment that H *weren't* on the line that passes through G and I.



To say that GH + HI = GI is to say that the journey from G to H and then to I is the same distance as the journey from G straight to I. That's nonsense unless H is on the road from G to I. Namely, H is between G and I.

If H isn't between G and I, then the side trip to H always takes longer. If H isn't between G and I, then GH + HI > GI. (> is the symbol from algebra for *greater than.*)

We have arrived at:

<u>Definition 1</u>: H is **between** G and I if and only if GH + HI = GI.

Some notes about definition 1:

♪#1: The word we're defining is in **boldface type**.

♪#2: Every definition in mathematics has the phrase "if and only if" in it. That means in the case of this definition that:

I) If H is between G and I then it's true that GH + HI = GI, and

II) If GH + HI = GI, then H is between G and I.

Either part of the definition implies the other part.

♪#3: Later on in geometry when we're doing a proof and we know, for example, that AB + BC = AC, then we can say, "B is between A and C" and give as a reason, "By the definition of between."

♪#4: Definitions are entirely optional. We could do all of geometry without using a single definition. However, we use definitions to *make our lives easier.* We could always talk about "the geometrical object consisting of three non-collinear points A, B and C and the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$," but don't you think it is a lot easier if we just say "triangle ABC"? Oops! We just made our second definition in geometry. We will call it Definition 2. It is the definition of triangle ABC, which we write in symbols as △ABC.

♪#5: Since we are introducing symbols, we will write "H is between G and I" as G-H-I.

♪#6: If G and H are the same point, then GH = 0 and G-H-I. You have more than one name, don't you? A point can have more than one name.

♪#7: Do you want to learn geometry? To make the whole thing a happy experience, here's a suggestion which has worked for many students over the years. Take out a sheet of paper and head it, "Definitions." There will be lots of definitions in geometry and it will be a lot easier if you copy them down as you encounter them. Then you will have them all in one place for easy reference. So far, your definitions page and symbols page would look like:

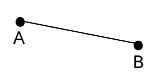
Definitions

Definition 1: H is between G and I if and only if GH + HI = GI.

Definition 2: <u>Triangle ABC</u> is defined as non-collinear points A, B and C and the line segments $\overline{AB}$, $\overline{BC}$ and $\overline{CA}$.

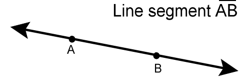
List of Symbols

$\overline{AB}$ is the notation for the line segment with endpoints A and B.



Line segment $\overline{AB}$ with endpoints A and B

$\overleftrightarrow{AB}$ is the notation for the line which contains A and B.



Line $\overleftrightarrow{AB}$ which passes through A and B

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